In [1]:

**import** numpy **as** np

**def** gradient\_descent(f,initial\_point,learning\_rate,iterations): point**=**initial\_point.copy()

history**=**[]

**for** \_ **in** range(iterations):

gradient**=**np.array([2**\***point[0],2**\***point[1]]) point**-=**learning\_rate**\***gradient

value**=**f(point[0],point[1])

history.append((point.copy(),value))

print("Values of points and objective function value are: {} and {}

**return** point,history

**def** objective\_function(x,y):

**return** x**\*\***2 **+** y**\*\***2 **+**4

initial\_point**=**np.array([1.0,1.0]) learning\_rate**=**0.1

iterations**=**100

minimum\_point,history**=**gradient\_descent(objective\_function,initial\_point,lea print("Minimum point(x,y):", minimum\_point)

print("Minimum value of the function:",objective\_function(**\***minimum\_point))

Values of points and objective function value are: [0.8 0.8] and 5.28

Values of points and objective function value are: [0.64 0.64] and 4.8192 Values of points and objective function value are: [0.512 0.512] and 4.524 288

Values of points and objective function value are: [0.4096 0.4096] and 4.3 3554432

Values of points and objective function value are: [0.32768 0.32768] and 4.2147483648

Values of points and objective function value are: [0.262144 0.262144] and 4.137438953472

Values of points and objective function value are: [0.2097152 0.2097152] a

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| nd 4.08796093022208 |  | | | | | |
| Values of points and | objective | function | value | are: | [0.16777216 | 0.16777216] |
| and 4.056294995342131 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.13421773 | 0.13421773] |
| and 4.036028797018964 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.10737418 | 0.10737418] |
| and 4.023058430092137 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.08589935 | 0.08589935] |
| and 4.014757395258967 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.06871948 | 0.06871948] |
| and 4.009444732965739 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.05497558 | 0.05497558] |
| and 4.006044629098073 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.04398047 | 0.04398047] |
| and 4.003868562622767 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.03518437 | 0.03518437] |

and 4.0024758800785705

Values of points and objective function value are: [0.0281475 0.0281475] a nd 4.001584563250285

Values of points and objective function value are: [0.022518 0.022518] and 4.001014120480183

Values of points and objective function value are: [0.0180144 0.0180144] a nd 4.0006490371073165

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| Values of points and | objective | function | value | are: | [0.01441152 | 0.01441152] |
| and 4.000415383748683 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.01152922 | 0.01152922] |
| and 4.000265845599157 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.00922337 | 0.00922337] |
| and 4.00017014118346 |  |  |  |  |  |  |
| Values of points and  nd 4.0001088903574145 | objective | function | value | are: | [0.0073787 0.0073787] a | |
| Values of points and | objective | function | value | are: | [0.00590296 | 0.00590296] |
| and 4.000069689828745 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.00472237 | 0.00472237] |
| and 4.000044601490397 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.00377789 | 0.00377789] |
| and 4.000028544953854 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.00302231 | 0.00302231] |
| and 4.000018268770467 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.00241785 | 0.00241785] |
| and 4.000011692013099 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.00193428 | 0.00193428] |
| and 4.000007482888383 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.00154743 | 0.00154743] |
| and 4.000004789048565 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.00123794 | 0.00123794] |
| and 4.000003064991081 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.00099035 | 0.00099035] |
| and 4.000001961594292 |  |  |  |  |  |  |
| Values of points and | objective | function | value | are: | [0.00079228 | 0.00079228] |

and 4.000001255420347

Values of points and objective and 4.000000803469022

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| function | value | are: | [0.00063383 | 0.00063383] | |
| function | value | are: | [0.00050706 | 0.00050706] | |
| function | value | are: | [0.00040565 | 0.00040565] | |
| function | value | are: | [0.00032452 | 0.00032452] | |
| function | value | are: | [0.00025961 | 0.00025961] | |
| function | value | are: | [0.00020769 | 0.00020769] | |
| function | value | are: | [0.00016615 | 0.00016615] | |
| function | value | are: | [0.00013292 | 0.00013292] | |
| function | value | are: | [0.00010634 | 0.00010634] | |
| function | value | are: | [8.50705917e-05 | | 8.50705 |

Values of points and objective and 4.000000514220174

Values of points and objective and 4.000000329100912

Values of points and objective and 4.0000002106245836

Values of points and objective and 4.000000134799733

Values of points and objective and 4.00000008627183

Values of points and objective and 4.000000055213971

Values of points and objective and 4.000000035336941

Values of points and objective and 4.000000022615643

Values of points and objective 917e-05] and 4.000000014474011

Values of points and objective function value are: [6.80564734e-05 6.80564 734e-05] and 4.000000009263367

Values of points and objective function value are: [5.44451787e-05 5.44451 787e-05] and 4.0000000059285545

Values of points and objective function value are: [4.3556143e-05 4.355614 3e-05] and 4.000000003794275

Values of points and objective function value are: [3.48449144e-05 3.48449 144e-05] and 4.000000002428336

Values of points and objective function value are: [2.78759315e-05 2.78759 315e-05] and 4.0000000015541355

Values of points and objective function value are: [2.23007452e-05 2.23007 452e-05] and 4.000000000994646

Values of points and objective function value are: [1.78405962e-05 1.78405 962e-05] and 4.0000000006365735

Values of points and objective function value are: [1.42724769e-05 1.42724 769e-05] and 4.000000000407407

Values of points and objective function value are: [1.14179815e-05 1.14179 815e-05] and 4.000000000260741

Values of points and objective function value are: [9.13438523e-06 9.13438 523e-06] and 4.000000000166874

Values of points and objective function value are: [7.30750819e-06 7.30750 819e-06] and 4.000000000106799

Values of points and objective function value are: [5.84600655e-06 5.84600 655e-06] and 4.0000000000683515

Values of points and objective function value are: [4.67680524e-06 4.67680 524e-06] and 4.0000000000437455

Values of points and objective function value are: [3.74144419e-06 3.74144 419e-06] and 4.000000000027997

Values of points and objective function value are: [2.99315535e-06 2.99315 535e-06] and 4.000000000017918

Values of points and objective function value are: [2.39452428e-06 2.39452 428e-06] and 4.000000000011467

Values of points and objective function value are: [1.91561943e-06 1.91561 943e-06] and 4.000000000007339

Values of points and objective function value are: [1.53249554e-06 1.53249 554e-06] and 4.000000000004697

Values of points and objective function value are: [1.22599643e-06 1.22599 643e-06] and 4.0000000000030065

Values of points and objective function value are: [9.80797146e-07 9.80797 146e-07] and 4.000000000001924

Values of points and objective function value are: [7.84637717e-07 7.84637 717e-07] and 4.000000000001231

Values of points and objective function value are: [6.27710174e-07 6.27710 174e-07] and 4.000000000000788

Values of points and objective function value are: [5.02168139e-07 5.02168 139e-07] and 4.0000000000005045

Values of points and objective function value are: [4.01734511e-07 4.01734 511e-07] and 4.000000000000322

Values of points and objective function value are: [3.21387609e-07 3.21387 609e-07] and 4.000000000000207

Values of points and objective function value are: [2.57110087e-07 2.57110 087e-07] and 4.000000000000132

Values of points and objective function value are: [2.0568807e-07 2.056880 7e-07] and 4.000000000000084

Values of points and objective function value are: [1.64550456e-07 1.64550 456e-07] and 4.000000000000054

Values of points and objective 365e-07] and 4.000000000000035

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| --- | --- | --- | --- | --- |
| function | value | are: | [1.31640365e-07 | 1.31640 |
| function | value | are: | [1.05312292e-07 | 1.05312 |
| function | value | are: | [8.42498333e-08 | 8.42498 |
| function | value | are: | [6.73998667e-08 | 6.73998 |
| function | value | are: | [5.39198933e-08 | 5.39198 |
| function | value | are: | [4.31359147e-08 | 4.31359 |
| function | value | are: | [3.45087317e-08 | 3.45087 |
| function | value | are: | [2.76069854e-08 | 2.76069 |
| function | value | are: | [2.20855883e-08 | 2.20855 |
| function | value | are: | [1.76684706e-08 | 1.76684 |

Values of points and objective 292e-07] and 4.000000000000022

Values of points and objective 333e-08] and 4.000000000000014

Values of points and objective 667e-08] and 4.000000000000009

Values of points and objective 933e-08] and 4.000000000000006

Values of points and objective 147e-08] and 4.0000000000000036

Values of points and objective 317e-08] and 4.000000000000003

Values of points and objective 854e-08] and 4.000000000000002

Values of points and objective 883e-08] and 4.000000000000001

Values of points and objective 706e-08] and 4.000000000000001

Values of points 765e-08] and 4.0

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| and | objective | function | value | are: | [1.41347765e-08 | 1.41347 |
| and | objective | function | value | are: | [1.13078212e-08 | 1.13078 |
| and | objective | function | value | are: | [9.04625697e-09 | 9.04625 |
| and | objective | function | value | are: | [7.23700558e-09 | 7.23700 |
| and | objective | function | value | are: | [5.78960446e-09 | 5.78960 |
| and | objective | function | value | are: | [4.63168357e-09 | 4.63168 |
| and | objective | function | value | are: | [3.70534686e-09 | 3.70534 |
| and | objective | function | value | are: | [2.96427748e-09 | 2.96427 |
| and | objective | function | value | are: | [2.37142199e-09 | 2.37142 |
| and | objective | function | value | are: | [1.89713759e-09 | 1.89713 |
| and | objective | function | value | are: | [1.51771007e-09 | 1.51771 |
| and | objective | function | value | are: | [1.21416806e-09 | 1.21416 |
| and | objective | function | value | are: | [9.71334446e-10 | 9.71334 |

Values of points 212e-08] and 4.0

Values of points 697e-09] and 4.0

Values of points 558e-09] and 4.0

Values of points 446e-09] and 4.0

Values of points 357e-09] and 4.0

Values of points 686e-09] and 4.0

Values of points 748e-09] and 4.0

Values of points 199e-09] and 4.0

Values of points 759e-09] and 4.0

Values of points 007e-09] and 4.0

Values of points 806e-09] and 4.0

Values of points

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 446e-10] and 4.0 |  | | | | | | |
| Values of points | and | objective | function | value | are: | [7.77067557e-10 | 7.77067 |
| 557e-10] and 4.0 |  |  |  |  |  |  |  |
| Values of points | and | objective | function | value | are: | [6.21654046e-10 | 6.21654 |
| 046e-10] and 4.0 |  |  |  |  |  |  |  |
| Values of points | and | objective | function | value | are: | [4.97323236e-10 | 4.97323 |
| 236e-10] and 4.0 |  |  |  |  |  |  |  |
| Values of points | and | objective | function | value | are: | [3.97858589e-10 | 3.97858 |
| 589e-10] and 4.0 |  |  |  |  |  |  |  |
| Values of points | and | objective | function | value | are: | [3.18286871e-10 | 3.18286 |
| 871e-10] and 4.0 |  |  |  |  |  |  |  |
| Values of points | and | objective | function | value | are: | [2.54629497e-10 | 2.54629 |
| 497e-10] and 4.0 |  |  |  |  |  |  |  |
| Values of points | and | objective | function | value | are: | [2.03703598e-10 | 2.03703 |
| 598e-10] and 4.0 |  |  |  |  |  |  |  |

Minimum point(x,y): [2.03703598e-10 2.03703598e-10] Minimum value of the function: 4.0

In [ ]: